Question	Scheme	Marks
1	$\sqrt[3]{(8-3x)^2} = \underline{(8)^{\frac{2}{3}}}\left(1-\frac{3}{8}x\right)^{\frac{2}{3}} = \underline{4}\left(1-\frac{3}{8}x\right)^{\frac{2}{3}}$ $\underline{(8)^{\frac{2}{3}}}$ or $\underline{4}$	**B1**
	$=\{4\}\left[1+\frac{2}{3}\left(-\frac{3}{8}x\right)+\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}\left(-\frac{3}{8}x\right)^2+\frac{(\frac{2}{3})(-\frac{1}{3})(-\frac{4}{3})}{3!}\left(-\frac{3}{8}x\right)^3+\ldots\right]$ or $=(8)^{\frac{2}{3}}+\frac{2}{3}(8)^{-\frac{1}{3}}(-3x)+\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(8)^{-\frac{4}{3}}(-3x)^2+\frac{(\frac{2}{3})(-\frac{1}{3})(-\frac{4}{3})}{3!}(8)^{-\frac{7}{3}}(-3x)^3+\ldots$	M1 A1
	$=4-x-\frac{1}{16}x^2-\frac{1}{96}x^3-\ldots$	A1A1
		(5)
		(5 marks)

Notes:

B1: $\underline{(8)^{\frac{2}{3}}}$ or $\underline{4}$ outside brackets then isw, or $\underline{(8)^{\frac{2}{3}}}$ or $\underline{4}$ as constant term in their binomial expansion.

M1: Expands $(..+kx)^{\frac{2}{3}}$ to give any 2 of the terms in x, x^2 or x^3 correct unsimplified, or achieves any two of the last three terms correct unsimplied by direct expansion.
A1: A fully correct unsimplified expansion.
A1: Any two of the final three terms correct and simplified.
A1: Fully correct, simplified expression.

Question	Scheme	Marks
2(a)	$\dfrac{1}{(2x+3)(3x+2)}=\dfrac{A}{2x+3}+\dfrac{B}{3x+2}\Rightarrow 1=A(3x+2)+B(2x+3)$	B1
	E.g. $x=-\dfrac{3}{2}\Rightarrow 1=-\dfrac{5}{2}A\Rightarrow A=...$ and $x=-\dfrac{2}{3}\Rightarrow 1=\dfrac{5}{3}B\Rightarrow B=...$	M1
	So $\dfrac{1}{(2x+3)(3x+2)}=\dfrac{-2}{5(2x+3)}+\dfrac{3}{5(3x+2)}$ (oe)	A1
		(3)
(b)	$(2x+3)(3x+2)\dfrac{dy}{dx}=5y\Rightarrow\dfrac{1}{y}\dfrac{dy}{dx}=\dfrac{3}{(3x+2)}-\dfrac{2}{(2x+3)}$	B1ft
	$\Rightarrow\displaystyle\int\frac{1}{y}\,dy=\int\frac{3}{3x+2}-\frac{2}{2x+3}\,dx\Rightarrow k\ln y=p\ln(3x+2)+q\ln(2x+3)\ (+c)$	M1
	$\ln y=\ln(3x+2)-\ln(2x+3)\ (+c)$ (oe)	A1ft
	So $\ln y=\ln\left(K\dfrac{3x+2}{2x+3}\right)$	M1
	$y=K\dfrac{3x+2}{2x+3}$	A1
		(5)
		(8 marks)

Notes:

(a)
B1: Correct statement of partial fraction form and multiplies through. May be implied if using cover-up rule – award if an answer of correct form is given.
M1: A correct method to find A and B. May be implied by one correct answer.
A1: Correct partial fraction expression. May be recovered in (b) if not explicitly stated in (a).

(b)
B1ft: Rearranges and uses their answer to part (a) to achieve an expression to a form ready for integration (may only see the integral step). Accept equivalents and follow through the A and B from (a).
M1: Integrates to achieve a form $k\ln y=p\ln(3x+2)+q\ln(2x+3)\ (+c)$. Constant of integration not needed for this mark. Accept equivalent forms, e.g. with constant multiples of the arguments such as $p\ln(9x+6)+q\ln(4x+6)$
A1: Correct integration for their equation. Follow through on their rearranged differential equation if slips have been made, as long as it is of the correct form. Constant of integration not needed.
M1: Uses a correct law of logarithms at least once to achieve both sides as single logarithms, and the constant of integration must be included.
A1: Correct answer, as scheme.

Question	Scheme	Marks
3	$\frac{\mathrm{d}u}{\mathrm{d}x} = \sec^2 x$ or $\mathrm{d}u = \sec^2 x\,\mathrm{d}x$ or $\mathrm{d}u = \frac{\mathrm{d}x}{\cos^2 x}$ oe	B1
	$\int_0^{\frac{\pi}{3}} \frac{1}{\cos^2 x + \sin x \cos x}\,\mathrm{d}x = \int_0^{\frac{\pi}{3}} \frac{1}{1+\tan x}\,\frac{\mathrm{d}x}{\cos^2 x} = \int_1^{1+\sqrt{3}} \frac{1}{u}\,\mathrm{d}u$	M1A1
	$\int \frac{k}{u}\,\mathrm{d}u \to k\ln u$ or $\int \frac{k}{1+u}\,\mathrm{d}u \to k\ln(1+u)$	M1
	$\int_1^{1+\sqrt{3}} \frac{1}{u}\,\mathrm{d}u = [\ln u]_1^{1+\sqrt{3}} = \ln(1+\sqrt{3}) - \ln 1$ OR $[\ln(1+\tan(x))]_0^{\frac{\pi}{3}} = \ln(1+\sqrt{3}) - \ln 1$	M1
	$= \ln(1+\sqrt{3})$	A1
		(6)
		(6 marks)

Notes:

B1: For a correct statement relating du and dx

M1: Applies the substitution to the integrand, achieving an integral in u only having used both $u = 1 + \tan x$ and replaced dx using their relationship between du and dx (limits not needed).

May see variations such as $u = 1 + \tan x = \frac{\cos x + \sin x}{\cos x} \Rightarrow u\cos^2 x = \cos^2 x + \sin x \cos x$ used to make the substitution, which is fine.

A1: For $\int \frac{1}{u}\,\mathrm{d}u$. Limits not needed for this mark.

M1: Correctly integrates something of the form $\frac{k}{u}$ or $\frac{k}{1+u}$

M1: Applies appropriate limits to a changed function. If in terms of u these should be 1 and $1+\sqrt{3}$, or if returning to x then 0 and $\frac{\pi}{3}$

A1: For $\ln(1+\sqrt{3})$

Question	Scheme	Marks
4(a)	$\frac{\mathrm{d}A}{\mathrm{d}t}=2-x$ stated or implied by working.	B1
	Cross sectional area is $A=\frac{1}{2}x^2\sin\left(\frac{\pi}{3}\right)=\frac{x^2\sqrt{3}}{4}$	B1
	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}t}\div\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{2-x}{"\frac{x\sqrt{3}}{2}"}$	M1
	$=\frac{4-2x}{x\sqrt{3}}$ oe	A1
		(4)
(b)	$V=3xA=\frac{3\sqrt{3}}{4}x^3\to\frac{\mathrm{d}V}{\mathrm{d}x}=\ldots$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}x}=\frac{9\sqrt{3}}{4}x^2$	A1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{9}{2}x(2-x)\Rightarrow\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right\|_{x=2.05}=\ldots$	M1
	$=-\frac{369}{800}$ so rate of decrease is $\frac{369}{800}(\mathrm{cm^3s^{-1}})$ Accept $\pm$awrt 0.461	A1
		(4)
		(8 marks)

Notes:

(a)

B1: States or implies by working that $\frac{\mathrm{d}A}{\mathrm{d}t}=2-x$

B1: A correct expression for the cross sectional area given.

M1: Applies the change rule with their $\frac{\mathrm{d}A}{\mathrm{d}t}$ and attempt at differentiation their area formula. May start with any acceptable version of the chain rule but must make $\frac{\mathrm{d}x}{\mathrm{d}t}$ the subject to gain this mark.

A1: Correct answer, accept equivalents and need not be simplified.

(b)

M1: Multiples their A by $3x$ and attempts to differentiate the result.

A1: Correct $\frac{\mathrm{d}V}{\mathrm{d}x}$

M1: Uses a correct chain rule to find the rate of change when $x = 2.05$. May find an expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ first and then substitute for x, or may evaluate both $\frac{\mathrm{d}V}{\mathrm{d}x}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$ at $x = 2.05$ and multiply the results, viz. $\frac{15129\sqrt{3}}{1600}\times-\frac{2}{41\sqrt{3}}$ or $16.377\ldots\times-0.0281\ldots$.

A1: Correct answer. Accept $\pm$awrt 0.461

Question	Scheme	Marks
5(a)	$\sin 3t = \sin(2t+t) = \sin 2t\cos t + \cos 2t\sin t$	**M1**
	$= (2\sin t\cos t)\cos t + (1-2\sin^2 t)\sin t$	**M1**
	$= 2\sin t(1-\sin^2 t) + \sin t - 2\sin^3 t = 3\sin t - 4\sin^3 t$ * cso	**A1***
		(3)
(b)	$\dfrac{dx}{dt} = 8\sin t\cos t$ (oe)	**B1**
	Area shaded $= \displaystyle\int_0^{\frac{\pi}{3}} 3\sin 3t \times "8\sin t\cos t" dt$	**M1**
	$= 24\displaystyle\int_0^{\frac{\pi}{3}} 3\sin^2 t\cos t - 4\sin^4 t\cos t\, dt$	**A1**
	$= 24\left[\dfrac{3\sin^3 t}{3} - \dfrac{4\sin^5 t}{5}\right]_0^{\frac{\pi}{3}}$	**M1** **A1ft**
	$= 24\left(\left(\dfrac{\sqrt{3}}{2}\right)^3 - \dfrac{4}{5}\left(\dfrac{\sqrt{3}}{2}\right)^5 - (0-0)\right) = \ldots \left(= \dfrac{18\sqrt{3}}{5}\right)$	**dM1**
	Proportion shaded is $\dfrac{\text{their area}}{\pi \times 3^2}$ or percentage shaded is $\dfrac{\text{their area}}{\pi \times 3^2} \times 100$	**M1**
	So percentage shaded is $\left(\dfrac{18\sqrt{3}}{45\pi} \times 100 = \right)$ 22.1% (3 s.f.)	**A1**
		(8)
		(11 marks)

Notes:

(a)
M1: Applies the compound angle formula to achieve $\pm\sin 2t\cos t \pm \cos 2t\sin t$
M1: Uses double angle formulae or compound angle formulae again to reach an expression with single angle arguments.
A1*cso: Applies $\cos^2 t = 1 - \sin^2 t$ and reaches correct expression, with no errors seen in working.

(b)
B1: Correct $\dfrac{dx}{dt}$ (need not be simplified).

M1: Sets up the integral required to find the area of the shaded area, using their $y\dfrac{dx}{dt}$. There must have been an attempt to differentiate x wrt t. (Limits not needed for this mark).

A1: Correct integrand in terms of single arguments (ie uses part (a) in correct expression to get to an integrable form) with correct limits, may be implied.

M1: Attempts the integration with $\sin^n t\cos t \to \pm k\sin^{n+1} t$ at least once.

A1ft: Correct integration of their integrand of the form $K\sin^2 t\cos t - M\sin^4 t\cos t$

$$K\sin^2 t\cos t - M\sin^4 t\cos t \to \frac{K}{3}\sin^3 t - \frac{M}{5}\sin^5 t$$

dM1: Applies limits to their integral and subtracts the either way round to find the area of the shaded shape. The zeroes from the lower limit may be implied. The trigonometric values must be evaluated (accept decimals) for this mark.

M1: Finds the proportion of the circle shaded using their area and Area of circle = $\pi\times 3^2 = 9\pi$

A1: Percentage covered is 22.1% to 3s.f.

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Alternative question 5 mark scheme.

Question	Scheme	Marks
5(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{k}{0.7t+1}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}t}=at-b$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{80\times0.7}{0.7t+1}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}t}=16t-38$	A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}t}\div\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{\text{their } 16t-38}{\text{their } \dfrac{56}{0.7t+1}}$	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{5}\left(t^2-\dfrac{53}{56}t-\dfrac{95}{28}\right)$	A1
		(4)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{5}\left(\left(t-\dfrac{53}{112}\right)^2-\left(\dfrac{53}{112}\right)^2-\dfrac{190}{56}\right)$ OR $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{1}{5}\left(2t-\dfrac{53}{56}\right)$	M1
	$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}\,x\ldots\dfrac{1}{5}\times-3.6167\ldots$ OR stationary point is at $t=\dfrac{53}{112}$ so is $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\ldots$	M1
	So steepest point has gradient awrt -0.723 to 3 s.f..	A1
		(3)
(c)	$\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\vert_{t=3}=\dfrac{31}{56}$	B1ft
	So angle to horizontal is $\arctan\left(\dfrac{31}{56}\right)=\ldots$	M1
	$=29.0°$ or 0.506 radians	A1
		(3)
		(10 marks)

Notes:

(a)

M1: Attempts both derivatives with respect to t. (Alt: eliminates t to find y in terms of x)

A1: Both derivatives correct. (Alt: correct derivative in terms of x)

M1: Applies $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}t}\div\dfrac{\mathrm{d}x}{\mathrm{d}t}$ (Alt: substitutes x back into their derivative)

A1: Correct unsimplified expression.

A1: Correct answer.

(b)

M1: Completes the square, or uses differentiation to find stationary point, or any other valid method to determine the steepest point (may be done using Cartesian equation).

M1: Extracts the minimum from their completed square form, or substitutes the t (or x if using Cartesian) from their derivative equated to zero back into $\frac{dy}{dx}$. **A1:** Correct answer, accept awrt.
(c) **B1ft**: Evaluates $\frac{dy}{dx}$ at $t=3$ (or $x=80\ln(3.1)$). Must be as given if (a) is correct, but follow through their a and b. **M1:** Applies arctan to their gradient at $t=3$. **A1:** Correct answer in degrees or radians (must be 3 s.f.)

5(a) ALT	$t=\frac{e^{x/80}-1}{0.7}\Rightarrow y=f(x)$	**M1**
	$y=8\left(\frac{e^{x/80}-1}{0.7}\right)^2-38\left(\frac{e^{x/80}-1}{0.7}\right)+100$ or $y=\frac{20}{49}\left(40e^{x/40}-213e^{x/80}+418\right)$ oe	**A1**
	$\frac{dy}{dx}=\frac{20}{49}\left(..e^{x/40}-..e^{x/80}\right)=\frac{20}{49}\left(..e^{2\ln(0.7t+1)}-..e^{\ln(0.7t+1)}\right)$	**M1** **A1**
	$\frac{dy}{dx}=\frac{1}{5}\left(t^2-\frac{53}{56}t-\frac{95}{28}\right)$	**A1**
		(4)

Notes:
(a) **M1**: Finds expression for t in terms of x and substitutes into equation for y **A1:** Correct expression in any form. **M1**: Attempts the differentiation with $e^{kx}\to..e^{kx}$ where .. is a constant and substitutes back in for x to get an expression in t only. **A1:** Correct unsimplified expression. **A1:** Correct answer.

Question	Scheme	Marks
6(a)	Two of $9+2\lambda=13+10\mu$ (1) $5+\lambda=-3-5\mu$ (2) $12+3\lambda=4+\mu$ (3)	M1
	E.g. (1) and (2) $\Rightarrow \lambda=..$ or $\mu=..$	M1
	$\lambda=-3$ or $\mu=-1$	A1
	Check in (3) : $12+3\times-3=3=4-1$, so they intersect.	B1
	$\overrightarrow{OX} = \begin{pmatrix}9\\5\\12\end{pmatrix}-3\begin{pmatrix}2\\1\\3\end{pmatrix}=\ldots$ or $\begin{pmatrix}13\\-3\\4\end{pmatrix}-\begin{pmatrix}10\\-5\\1\end{pmatrix}=\ldots$	M1
	$=\begin{pmatrix}3\\2\\3\end{pmatrix}$	A1
		(6)
(b)	$\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{XB}=\begin{pmatrix}9\\5\\12\end{pmatrix}+\begin{pmatrix}10\\-5\\1\end{pmatrix}$ or $\overrightarrow{OP}=\overrightarrow{OB}+\overrightarrow{XA}=\begin{pmatrix}13\\-3\\4\end{pmatrix}+3\begin{pmatrix}2\\1\\3\end{pmatrix}$	M1
	$=\begin{pmatrix}19\\0\\13\end{pmatrix}$	A1
		(2)
(c)	$\lvert\overrightarrow{XA}\rvert=\sqrt{6^2+3^2+9^2}$ or $\lvert\overrightarrow{XB}\rvert=\sqrt{10^2+(-5)^2+1^2}$ OR $\overrightarrow{XP}.\overrightarrow{BA}=\begin{pmatrix}16\\-2\\10\end{pmatrix}.\begin{pmatrix}-4\\8\\8\end{pmatrix}=\ldots$	M1
	$\lvert\overrightarrow{XA}\rvert=\sqrt{126}$ and $\lvert\overrightarrow{XB}\rvert=\sqrt{126}$ hence a rhombus as sides are equal.* OR $\ldots=-64-16+80=0$ hence diagonals are perpendicular so a rhombus*	A1*
		(2)
(d)	Assume it is a square, hence the sides are perpendicular / diagonals are equal length.	M1
	But $\overrightarrow{XA}.\overrightarrow{XB}=\begin{pmatrix}6\\3\\9\end{pmatrix}.\begin{pmatrix}10\\-5\\1\end{pmatrix}=\ldots$ or $XP^2=16^2+2^2+10^2=\ldots$ $BA^2=4^2+8^2+8^2=\ldots$	M1

	$= 60 - 15 + 9 = 54 \neq 0$ hence sides are not perpendicular, a contradiction; or $XP^2 = 360 \neq 144 = BA^2$, a contradiction. So *XAPB* is not a square.*	**A1***
		(3)
		(13 marks)

Notes:

(a)

M1: Writes down any two correct equations in λ and μ.

M1: Attempts to solve any two of the equations to find one of λ or μ

A1: Either $\lambda = -3$ or $\mu = -1$ found.

B1: Verifies both values in third equation, or uses both in the equations of the lines and deduces intersects both equal.

M1: Uses either λ or μ in appropriate equation to find $\overrightarrow{OX}$

A1: Correct point X, accept as vector or coordinates.

ALT for first four marks

M1: Attempts at least one value for λ and μ in each equation.

M1: Identifies a pair of λ and μ for which the equations give the same point.

A1: Correct value for μ or λ

B1: Correct values used for both μ and λ with deduction the lines intersect.

(b)

M1: A correct equation for $\overrightarrow{OP}$ used - see scheme.

A1: Correct point, as position vector or coordinates.

(c)

M1: Correct method for at least one side length. May scale direction vector, but must be appropriate scaling. Alternatively may attempt dot product of diagonals or any other valid classification for a rhombus.

A1*: Both side lengths evaluated correctly with conclusion shape is a rhombus. Alternatively dot product shown zero with reference to being right angles and conclusion shape is a rhombus.

(d)

M1: Sets up the proof by identifying a feature other than side lengths are equal that is to be investigated. Usually that to be a square the sides must be perpendicular but other assumptions are possible (e.g. the diagonals must be equal length), which are equally fine.

M1: Finds the scalar product, or other appropriate feature (e.g. both diagonal lengths)

A1: Correctly deduces sides not perpendicular (or equivalent for their method) and hence a contradiction, so not a square.

Question	Scheme	Marks
7(a)	$\int e^{2x}\cos 2x\,dx = e^{2x}\times\frac{1}{2}\sin 2x - \int 2e^{2x}\times\frac{1}{2}\sin 2x\,dx$ or $\int e^{2x}\cos 2x\,dx = \frac{1}{2}e^{2x}\times\cos 2x - \int \frac{1}{2}e^{2x}\times -2\sin 2x\,dx$	**M1**
	$= e^{2x}\times\frac{1}{2}\sin 2x - \left(e^{2x}\times -\frac{1}{2}\cos 2x - \int 2e^{2x}\times -\frac{1}{2}\cos 2x\,dx\right)$ or $= \frac{1}{2}e^{2x}\times\cos 2x + \left(\frac{1}{2}e^{2x}\times\sin 2x - \int \frac{1}{2}e^{2x}\times 2\cos 2x\,dx\right)$	**M1** **A1**
	$\Rightarrow 2\int e^{2x}\cos 2x\,dx = \frac{1}{2}e^{2x}(\sin 2x + \cos 2x)$	**M1**
	$\Rightarrow \int e^{2x}\cos 2x\,dx = \frac{1}{4}e^{2x}(\sin 2x + \cos 2x) + c$ *	**A1***
		(5)
(b)	$\text{Volume} = \pi\int_0^{\frac{\pi}{2}} (e^x\cos x)^2\,dx$	**B1**
	$(e^x\cos x)^2 = e^{2x}\cos^2 x = \frac{1}{2}e^{2x}(1+\cos 2x)$	**M1**
	$\int \frac{1}{2}e^{2x}(1+\cos 2x)\,dx = \frac{1}{4}e^{2x} + \frac{1}{2}\int e^{2x}\cos 2x\,dx$	**M1**
	$= \frac{1}{4}e^{2x} + \frac{1}{8}e^{2x}(\sin 2x + \cos 2x)\ (+c)$	**A1**
	$\int_0^{\frac{\pi}{2}} \frac{1}{2}e^{2x}(1+\cos 2x)\,dx = \left[\frac{1}{4}e^{2x} + \frac{1}{8}e^{2x}(\sin 2x + \cos 2x)\right]_0^{\frac{\pi}{2}}$ $= \left(\frac{1}{8}e^{\pi}(2+\sin\pi+\cos\pi) - \frac{1}{8}e^{0}(2+\sin 0+\cos 0)\right)$	**M1**
	$\text{Volume} = \frac{\pi}{8}(e^{\pi} - 3)$	**A1**
		(6)
		(11 marks)

Notes:

(a)

M1: Attempts integration by parts either way around. Allow slips in signs or missing coefficients when differentiating/integrating the trigonometric terms, so accept anything of the form $\pm ke^{2x}\sin 2x \pm m\int e^{2x}\sin 2x\,dx$ where k is 1 or $\frac{1}{2}$ OR $\pm ke^{2x}\cos 2x \pm m\int e^{2x}\sin 2x\,dx$ where k is 1 or $\frac{1}{2}$ and m is 1 or 2

M1: Attempts parts a second time in the same direction. Same criteria as for first M. **A1:** Correct result of integrating twice by parts. Watch for double sign errors, which will gain A0. **M1:** Gathers the $\int e^{2x}\cos 2x\,dx$ terms in an attempt to make it the subject of their equation. **A1*:** Correct result with no errors in their working.
(b) **B1:** For a correct statement of the volume, with correct limits and including the π, seen or implied by subsequent working. The π may be included later, and is only needed for the first and last marks in this part. **M1:** Squares the integrand and attempts double angle formula to set up the integral. Accept $\frac{1}{2}e^{2x}\left(\pm 1 \pm \cos 2x\right)$ **M1:** Attempts the integration, $e^{2x} \to \frac{1}{2}e^{2x}$ and sets up second integral from part (a) **A1:** Correct integration **M1:** Applies the limits 0 and $\frac{\pi}{2}$ to their integral. **A1**: Correct exact answer. Accept equivalents but must be exact with trig terms evaluated.

Question	Scheme	Marks
8(a)	$\underline{11\left(1-2\frac{\mathrm{d}y}{\mathrm{d}x}\right)\mathrm{e}^{x-2y}}+5\underline{\underline{\left(y^2+2xy\frac{\mathrm{d}y}{\mathrm{d}x}\right)}}=\underline{10x}$	M1<u>A1</u> <u>B1</u>
	At $P(2,1)$, $11\left(1-2\frac{\mathrm{d}y}{\mathrm{d}x}\right)\mathrm{e}^{2-2}+5\left(1+4\frac{\mathrm{d}y}{\mathrm{d}x}\right)=20\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$	dM1
	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{(2,1)}=-2$ *	A1*
		(5)
(b)	Gradient of normal is $m_N=\frac{-1}{-2}\left(=\frac{1}{2}\right)$ so equation is $y-1="m_N"(x-2)$	M1
	Normal is $y-1=\frac{1}{2}(x-2)$ oe	A1
	Meets curve again when $11\mathrm{e}^{2y-2y}+5(2y)y^2=1+5(2y)^2$ or $11\mathrm{e}^{x-x}+5x\left(\frac{1}{2}x\right)^2=1+5x^2$	M1
	$10y^3-20y^2+10=0$ or $x^3-4x^2+8=0$ oe	A1
	$\Rightarrow(y-1)(y^2-y-1)=0$ or $(x-2)(x^2-2x-4)=0$	M1
	$y=\frac{1\pm\sqrt{1^2-4\times1\times(-1)}}{2(1)}=\ldots$ or $x=\frac{2\pm\sqrt{(-2)^2-4\times1\times(-4)}}{2(1)}=\ldots$	M1
	So meets curve again at $\left(1+\sqrt{5},\frac{1+\sqrt{5}}{2}\right)$ and $\left(1-\sqrt{5},\frac{1-\sqrt{5}}{2}\right)$	A1A1
		(8)
		(13 marks)

Notes:

(a)

M1: Attempts implicit differentiation of the given equation to include a term $kxy\frac{\mathrm{d}y}{\mathrm{d}x}$ or $k\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{e}^{\cdots}$ where … is $x-2y$ or just $-2y$

A1: Correct differentiation of the e^{x-2y} term and the RHS. Accept as per scheme or via $11\mathrm{e}^{x-2y}=11\mathrm{e}^{x}\mathrm{e}^{-2y}\rightarrow 11\mathrm{e}^{x}\mathrm{e}^{-2y}+11\mathrm{e}^{x}\times-2\mathrm{e}^{-2y}\frac{\mathrm{d}y}{\mathrm{d}x}$ or via quotient rule.

B1: For $xy^2\rightarrow y^2+2xy\frac{\mathrm{d}y}{\mathrm{d}x}$

dM1: Substitutes $x=2$ and $y=1$ into the equation and rearranges to find $\frac{dy}{dx}$. May first find $\frac{dy}{dx}=\frac{10x-5y^2-11e^{x-2y}}{10xy-22e^{x-2y}}$ and then substitute values, which is fine. Depends on first M.

A1*: Achieves $\frac{dy}{dx}=-2$ **by cso**. Do not allow if mistakes have been made in earlier work – e.g. if a y is missing they will get the same answer, but it is A0.

(b)

M1: Attempts to find the equation of the normal, using the answer given in (a) and the point $P(2,1)$. Accept $\frac{\pm 1}{m_T}$ for the gradient (so $y-1=\pm\frac{1}{2}(x-2)$ gains M1.

A1: Correct equation of the normal, $x-2y=0$ or any equivalent and isw.

M1: Substitutes for y or x from equation of normal back into the equation of the curve.

A1: A correct cubic in just x or y. The exponential must have been correctly dealt with for this mark. Accept equivalents, not necessarily all on one side, so e.g. $10y^3+10=20y^2$ is fine.

M1: Attempts to take out $(y-1)$ or $(x-2)$ as a factor from their cubic, as appropriate.

M1: Solves the resulting quadratic to find either the y values or the x values for their equation. Should use the formula or completion of the square. Accept attempts at factorisation only if their quadratic does factorise.

A1: Allow for either both x or y coordinates correct, or for one correct pairing of x and y coordinates.

A1: Both sets of coordinates correct, and no others. Accept $\left(1\pm\sqrt{5},\frac{1\pm\sqrt{5}}{2}\right)$ unless it is clear the student has four points in mind.